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Active control of waves in a Timoshenko beam

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Abstract

Active control of (bending) waves in Timoshenko beams is modeled and tackled. A discussion about the possible wave solutions for harmonically excited Timoshenko beams and their control by either forces or moment pairs is presented. It is also shown that the adoption of an extra control load allows to a minimum force wave cancellation control strategy to be developed with important advantages, when compared to the strict wave cancellation approach. The approach described in this paper was employed in the analysis of infinite beams; however, it is directly applicable to the case of limited beams with the controller blocking the passage from a source region to a region that is to be shielded.

Keywords: Control; Waves; Timoshenko; Beam

1. Introduction

Many practical engineering structures are conceived as assemblies of relatively slender elements that can be modeled as one-dimensional mechanical waveguides. Beams are important both as fundamental structural elements and as simple global models for some slender bodies.

Much attention has been devoted to their dynamic behavior. In particular, active control of beam vibrations is becoming an important field of research, having in view the possible engineering applications. Analyzed mainly from the modal point of view, dynamic control can also be handled through the wave propagation approach. This has special appeal because of its local character and better suitability to transient control at high modal density ranges. When sources are located far from regions to be protected, the wave control could be used to block or trap the vibrational energy flow. Some authors have adopted this approach in recent works (Mace, 1987; Elliot and Billet, 1993; Brennan et al., 1995; Pan and Hansen, 1995; Gardonio and Elliott, 1996; Chen et al., 1997; Hirami, 1997).

Physical models of flexural wave propagation in beams are developed in order to implement such a control. Special attention was devoted to the Euler–Bernoulli beam model, which is adequate to represent a

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fair range of structural elements. Some authors of the likes of Mace (1984) and Mead (1994), dedicated their effort to the wave reflection mechanism. Less frequently, the wave control of Timoshenko beams has also been investigated (Hagedorn and Schmidt, 1989; Tong et al., 1995; Farghali and Gadelrab, 1995; Houmat, 1995; Karlson, 1996; Lueschen et al., 1996; Corn et al., 1997).

The most common wave control strategy is the wave suppression. Other concepts, such as the maximization of power absorbed by the control forces or the minimization of the total power supplied to the beam by the original disturbing loads, have also been proposed. These three strategies are compared by Brennan et al. (1995) for the Euler–Bernoulli beam model.

In the present work, the Timoshenko beam model is adopted, due to its more complete representation of the beam behavior, especially at higher frequencies, which are the most important, for example, for problems in the acoustical range. The wave suppression strategy, as shown in Elliott and Billet (1993), is addressed due to its performance, straightforward results and low information requirements about the excitation sources. Some authors indicate the power control strategies as the best ones, mainly from the results in sound control, but they depend on rich information about the disturbing source, which may not be available in many cases. Also, one must remember the many practical differences between loudspeakers and force actuators, which justify the search for alternative concepts.

In this work, two actuator array models were developed: one with force actuators and another with pairs of moment actuators, as used by Gardonio and Elliott (1996). This configuration is of special interest because of the recent developments of moment actuators made of piezoceramic materials, as presented by Gibbs and Fuller (1992) and their similarity to biomechanical, muscular actuators. A simple construction may adapt a linear actuator to produce a self canceling moment pair, by projecting two *apophisae* out of the beam axis (Nagaya, 1995).

While a time domain control scheme, presented by Carvalho (1998), would be directly amenable to a transient wave control experimental setup with digital signal processing, much insight is gained through the preliminary frequency domain analysis presented here.

2. Equations of motion

The coupled differential equations for small amplitude transverse vibration of a uniform Timoshenko beam with constant cross-section may be written in the form

$$\rho A \frac{\partial^2 y}{\partial t^2} - KAG\left(\frac{\partial^2 y}{\partial x^2} - \frac{\partial \psi}{\partial x}\right) = p, \tag{1}$$

$$EI \frac{\partial^2 \psi}{\partial x^2} + KAG\left(\frac{\partial y}{\partial x} - \psi\right) - J\frac{\partial^2 \psi}{\partial t^2} = \mu, \tag{2}$$

where y is the transversal deflection and ψ is the rotation of the cross-section. E denotes Young's modulus, G, the shear modulus, I, the area moment of inertia, J, the mass moment of inertia, A, the cross-sectional area, ρ, the density and K, the shear factor, which depends on the shape of the beam cross-section. Also, p and μ are, respectively, the external distributed force and moment loads. Fig. 1 shows a (infinite) Timoshenko beam with flexural waves generated by a concentrated force load $p = f(t)\delta(x - x_0)$ or by a moment $\mu = M(t)\delta(x - x_0)$.

The bending moment T and shear force Q transmitted through an arbitrary section of the beam may be expressed by

$$T = -EI\frac{\partial \psi}{\partial x} = EI\left(\frac{\rho}{KG}\frac{\partial^2 y}{\partial t^2} - \frac{\partial^2 y}{\partial x^2}\right), \tag{3}$$

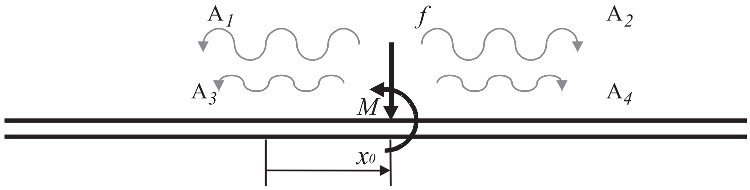

Fig. 1. Positive- and negative-propagating (or evanescent) waves generated by either a moment or a force in a position x_0.

$$Q = KAG\left(\frac{\partial y}{\partial x} - \psi\right) = EI\frac{\partial^2\psi}{\partial x^2} - J\frac{\partial^2\psi}{\partial t^2}. \tag{4}$$

Eqs. (1) and (2) may be combined for y resulting in the single Timoshenko beam dynamic equation

$$EI\frac{\partial^4 y}{\partial x^4} - \left(\frac{\rho EI}{KG} + J\right)\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 y}{\partial t^2}\right) + A\rho\frac{\partial^2 y}{\partial t^2} + \frac{\rho J}{KG}\frac{\partial^4 y}{\partial t^4} = -\frac{EI}{KAG}\frac{\partial^2 p}{\partial x^2} + p + \frac{J}{KAG}\frac{\partial^2 p}{\partial t^2} + \frac{\partial\mu}{\partial x}. \tag{5}$$

For concentrated loads, it is easier to use the load-free equation

$$EI\frac{\partial^4 y}{\partial x^4} - \left(\frac{\rho EI}{KG} + J\right)\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 y}{\partial t^2}\right) + A\rho\frac{\partial^2 y}{\partial t^2} + \frac{\rho J}{KG}\frac{\partial^4 y}{\partial t^4} = 0 \tag{6}$$

with appropriate jump conditions.

3. Solutions

If a concentrated harmonic load, either force or moment, is applied, at any point, to the (infinite) beam, four free flexural waves will emanate from this point. Harmonic solutions

$$y(x,t) = y(x)e^{i\omega t} \tag{7}$$

are assumed for Eq. (6) yielding

$$EI\frac{\partial^4 y}{\partial x^4} + \left(\frac{\rho EI}{KG} + J\right)\omega^2\frac{\partial^2 y}{\partial x^2} - A\rho\omega^2 y + \frac{\rho J}{KG}\omega^4 y = 0 \tag{8}$$

with the general solution

$$y(x) = A_1e^{\eta_1 x} + A_2e^{\eta_2 x} + A_3e^{\eta_3 x} + A_4e^{\eta_4 x}, \tag{9}$$

where η_i is found by solving the associated fourth-order characteristic equation

$$\eta^4 + \beta\eta^2 + \alpha\eta = 0, \tag{10}$$

with

$$\beta = \rho\omega^2\left(\frac{1}{KG} + \frac{1}{E}\right), \tag{11}$$

$$\alpha = \frac{\rho}{E}\left(\frac{\rho\omega^4}{KG} - \frac{A\omega^2}{I}\right). \tag{12}$$

While solving Eq. (10), two different families of solution occur, depending on the values of α and β, which combine to give the non-dimensional parameter

$$R = \frac{\omega^2 J}{AKG}. \tag{13}$$

For $R < 1$, one imaginary pair and two real and symmetric roots occur. In this case, the well-known solution leads to two pairs of propagating and evanescent waves. These solutions have the same nature as those found for the Euler–Bernoulli model.

If $R > 1$, then two pairs of imaginary roots appear. The corresponding solutions are two pairs of waves, propagating at two different speeds, referred to as two different *propagation modes*. Corn et al. (1997) describe this behavior in a modal approach. Such a solution has no near field, as can be seen in Figs. 2 and 3.

The case $R = 1$ is a limiting case, not important for a detail consideration here. It is relevant to point out that the region $R \geqslant 1$ corresponds to high frequencies, with high bending and low shearing rigidity designs. Such properties are more easily found in special materials, such as composites, like that studied by Farghaly and Gadelrab (1995). For example, in a steel beam with the following characteristics: $b = 0.1$ m, $h = 0.35$ m, $K = 0.75$, $G = 7.7 \times 10^{10}$ N/m^2, $\rho = 7.8 \times 10^3$ kg/m^3, where b is the width and h is the height of the solid rectangular cross-section, this condition would be reached for 4.28 kHz. For the simulations presented here, this very stiff beam was chosen.

It must be observed that $R = 0$ alone does not ensure an approximation between Timoshenko and Euler–Bernoulli models.

Also interesting to observe is that Eqs. (1) and (2) have a peculiar solution when the product *EI* becomes very large when compared with the quantity *KAG*. In this case, a mode of pure rotation of the beam cross-section takes place with no transverse displacement. Here, the beam degenerates in a sheaf of frictionless compressible fibers, governed by

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\rho}{E}\frac{\partial^2 \psi}{\partial t^2} = \frac{\mu}{EI}. \tag{14a}$$

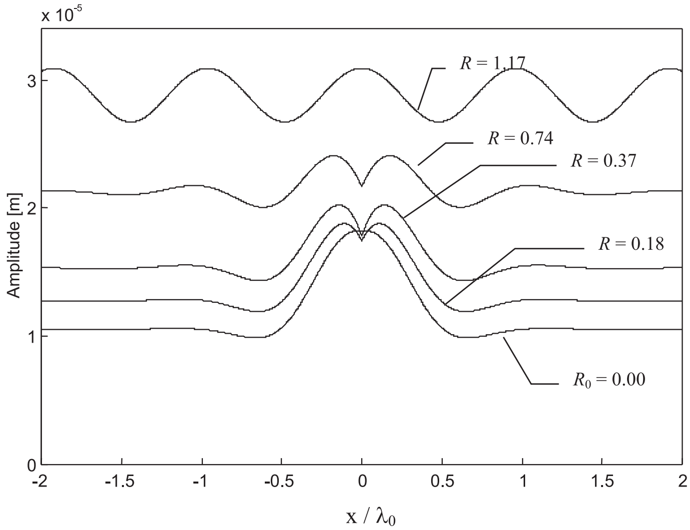

Fig. 2. Response amplitudes of Timoshenko beams for unit concentrated force loading at $x = 0$. The wavelength λ_0 is for $R = R_0$.

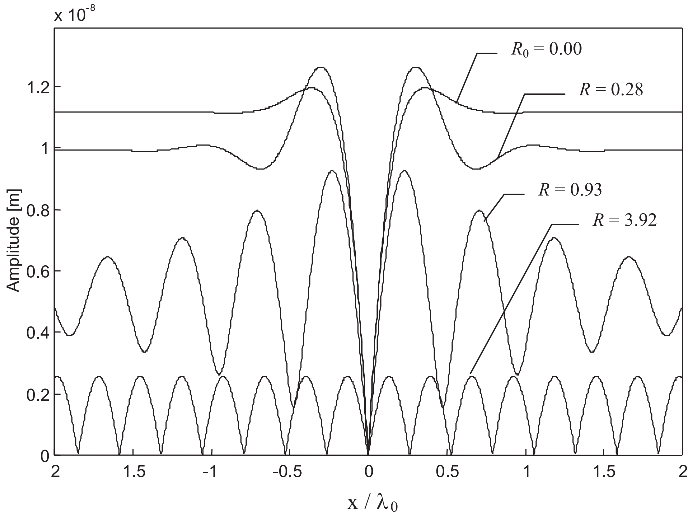

Fig. 3. Amplitude response of Timoshenko beams for unit concentrated moment loading at $x = 0$. The wavelength λ_0 is for $R = R_0$.

The associated propagating speed c_{R} is (fiber tensile waves)

$$c_{\mathrm{R}} = \sqrt{\frac{E}{\rho}}. \tag{14b}$$

Another particular situation arises when the bending stiffness EI is very large, when compared with both quantities KAG and J. In this case, Eqs. (1) and (2) become uncoupled, and a propagating wave mode of pure shear of the beam cross-section takes place:

$$\frac{\partial^2 y}{\partial x^2} - \frac{\rho}{KG}\frac{\partial^2 y}{\partial t^2} = \frac{p}{KG}. \tag{15a}$$

This equation has an associated propagating speed c_{s} expressed by

$$c_{\mathrm{s}} = \sqrt{\frac{KG}{\rho}}. \tag{15b}$$

The solutions (14) and (15) show that at limit conditions, two propagating (not evanescent) modes can exist together in the general Timoshenko solution.

4. Jump conditions

For an infinite Timoshenko beam, as shown in Fig. 1, the following matching conditions prevail in the section where concentrated loads (force f and moment M) are applied:

$$\begin{cases} y^+ = y^-, \\ \psi^+ = \psi^-, \\ M = T^+ - T^- = EI\Big[-\Big(\frac{\partial^2 y^+}{\partial x^2}\Big) + \Big(\frac{\partial^2 y^-}{\partial x^2}\Big) + \frac{\rho}{KG}\Big(\Big(\frac{\partial^2 y^+}{\partial t^2}\Big) - \Big(\frac{\partial^2 y^-}{\partial t^2}\Big)\Big)\Big], \\ f = Q^+ - Q^- = KAG\Big[(\psi^+ - \psi^-) - \Big(\frac{\partial y^+}{\partial x} - \frac{\partial y^-}{\partial x}\Big)\Big], \end{cases} \tag{16}$$

where $T(Q)$ is the internal bending moment (shear force) and $+$ $(-)$ means to the right (left) of the cross-section where the corresponding concentrated load is applied.

By eliminating ψ and imposing the time harmonic loading, one can finally obtain

$$\begin{cases} y^+ - y^- = 0, \\ \frac{\partial y^+}{\partial x} - \frac{\partial y^-}{\partial x} = -\frac{f}{KAG}, \\ \left(\frac{\partial^2 y^+}{\partial x^2}\right) - \left(\frac{\partial^2 y^-}{\partial x^2}\right) = -\frac{M}{EI}, \\ \left(\frac{\partial^3 y^+}{\partial x^3}\right) - \left(\frac{\partial^3 y^-}{\partial x^3}\right) = \frac{f}{EI}\left(1 + \frac{EI\rho\omega^2}{K^2G^2A}\right). \end{cases} \tag{17}$$

On substituting the solution of Eq. (9) in Eq. (17), the resulting system of equations can be written as

$$[C(l)]\{A\} = \{B\}. \tag{18}$$

For a position $x = l$, the beam (normalized) inverse compliance matrix is

$$[C(l)] = \begin{bmatrix} e^{\eta_1 l} & -e^{-\eta_1 l} & e^{\eta_2 l} & -e^{-\eta_2 l} \\ \eta_1 e^{\eta_1 l} & \eta_1 e^{-\eta_1 l} & \eta_2 e^{\eta_2 l} & -\eta_2 e^{-\eta_2 l} \\ \eta_1^2 e^{\eta_1 l} & -\eta_1^2 e^{-\eta_1 l} & \eta_2^2 e^{\eta_2 l} & -\eta_2^2 e^{-\eta_2 l} \\ \eta_1^3 e^{\eta_1 l} & \eta_1^3 e^{-\eta_1 l} & \eta_2^3 e^{\eta_2 l} & \eta_2^3 e^{-\eta_2 l} \end{bmatrix}, \tag{19}$$

the amplitude vector is

$$\{A\} = \{A_1 \quad A_2 \quad A_3 \quad A_4\}^{\mathrm{T}}, \tag{20}$$

and the load vector results in

$$\{B\} = \begin{Bmatrix} 0 \\ \left(\frac{f}{KA}\right) \\ \left(\frac{M}{EI}\right) \\ -\frac{f}{EI}\left(1 + \frac{EI\rho\omega^2}{K^2G^2A}\right) \end{Bmatrix}. \tag{21}$$

Then, if a load, either force or moment, is applied at a particular section $x = l$ on a beam, the coefficients A_i for the solution in Eq. (9) are obtained from

$$\{A\} = [C(l)]^{-1}\{B\}. \tag{22}$$

Finally, to calculate the effect of an array of external loads applied along the length of the beam, the solution superposition technique will be used as the system is linear.

5. Active control

Flexural propagating wave control by cancellation consists of prescribing a load array (of forces f_i or moments M_i and their application points) that eliminates the propagating disturbance, when it crosses the array. In this approach, the waves generated by the control loads cancel the incident propagating wave. As each load of the array generates four flexural waves, conditions are included in order to avoid or minimize side effects, which could be considered as a propagating spillover.

Let an external disturbance generate a progressive harmonic wave $D(x,t) = A_{\mathrm{in}} e^{i(kx-\omega t)}$ with wavenumber k and frequency ω. Then, in this expression, ω being imposed, k can be evaluated in correspondence to one of the roots of Eq. (10), as a function of the beam's mechanical properties.

Without loss of generality, let $A_{\mathrm{in}} = 1$ which, considering the four possible waves in Eq. (9), may be seen as part of the disturbance amplitude vector:

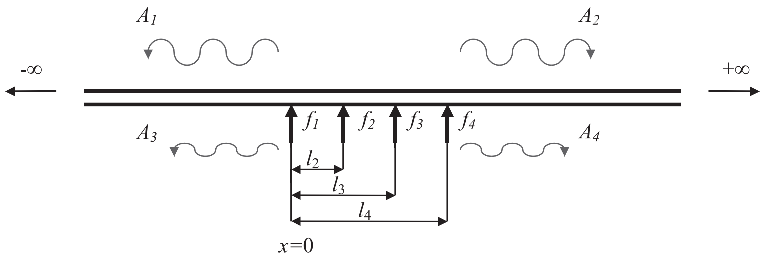

Fig. 4. Force array applied to an infinite beam and the waves generated by these forces out of the array. For simplicity, l_1 was made zero.

$$\{D(x,t)\} = \{0 \quad A_{\rm in} \quad 0 \quad 0\}^{\rm T} {\rm e}^{{\rm i}(kx-\omega t)} = \{0 \quad 1 \quad 0 \quad 0\}^{\rm T} {\rm e}^{{\rm i}(kx-\omega t)}. \tag{23}$$

5.1. Force array

Considering the control conditions of an infinite beam through the application of force actuators (Fig. 4), it can be verified that four independent forces are needed to impose the amplitudes of the four outgoing waves. Calling $\{A\}_i$ as the wave amplitude vector generated by a unit control force applied in $x = l_i$, one gets

$$\{A\}_i = [C(l_i)]^{-1}\{B\}_i, \tag{24}$$

where $\{B\}_i$ (see Eq. (21)) is the corresponding load vector applied when $x = l_i$. In the case of a control force array,

$$\{B\}_i = \left\{ \begin{array}{c} 0 \\ \left(\frac{1}{KA}\right) \\ 0 \\ \left[-\frac{1}{EI}\left(1 + \frac{EI\rho\omega^2}{K^2G^2A}\right)\right] \end{array} \right\}. \tag{25}$$

Then, the control waves generated by the four force array applied in l_1, l_2, l_3 and l_4 would be

$$\{A\}_{\rm control} = f_1\{A\}_1 + f_2\{A\}_2 + f_3\{A\}_3 + f_4\{A\}_4 = \sum_{i=1}^{4} f_i\{A\}_i \tag{26a}$$

or

$$\{A\}_{\rm control} = \sum_{i=1}^{4} f_i[C(l_i)]^{-1}\{B\}_i. \tag{26b}$$

5.2. Cancellation control

In the control strategy based on cancellation criteria, the addition $\{A\}_{\rm control}$ represents the four outgoing wave (complex) amplitudes or the amplitude vector generated by the control forces and should cancel the external disturbance $\{D\}$. Four levels of fulfillment may be the following:

1. The positive propagating disturbance wave stops in the control array. In this case, just one control force is enough but three other progressive and regressive waves will be generated out of the array as side effects. The nature of these waves depends on the value of R.

2. A pair of propagating waves is canceled in the control array. In this case, two control forces will be demanded and two other (either evanescent or propagating) waves would be produced out of the array.

3. A pair of propagating waves stops and one of the others, a progressive or regressive (either evanescent or propagating) wave, would also be canceled. In this case, three control forces will be demanded. This case is very interesting, having in mind the reduction of sensor contamination by regressive waves.

4. All the waves coming out of the array are canceled by a four force array and no side effects will appear. This complete cancellation control implies

$$\{A\}_{\text{control}} = \sum_{i=1}^{4} f_i\{A\}_i = -\{D\} = \{0 \quad -1 \quad 0 \quad 0\}^{\mathrm{T}} \tag{27}$$

resulting in

$$\left[\{A\}_1\{A\}_2\{A\}_3\{A\}_4\right]\{f\} = [A]\{f\} = \{0 \quad -1 \quad 0 \quad 0\}^{\mathrm{T}}, \tag{28}$$

where

$$\{f\} = \{f_1 \quad f_2 \quad f_3 \quad f_4\}^{\mathrm{T}}. \tag{29}$$

Therefore, the control forces $\{f\}$ can be evaluated by

$$\{f\} = [A]^{-1}\begin{Bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{Bmatrix}. \tag{30}$$

The results of such a complete control scheme applied to the example beam are illustrated ahead, in Fig. 8(d). If less than four forces are applied, i.e., if only a part of the generated waves is controlled, then a correspondingly reduced number of right-hand side conditions in Eq. (28) can be satisfied.

In Fig. 5, results for a single force are exemplified, and worth noticing is the standing wave pattern created by the regressive and incident (disturbance) waves. The near field details can also be seen as a function of the ratio x/λ.

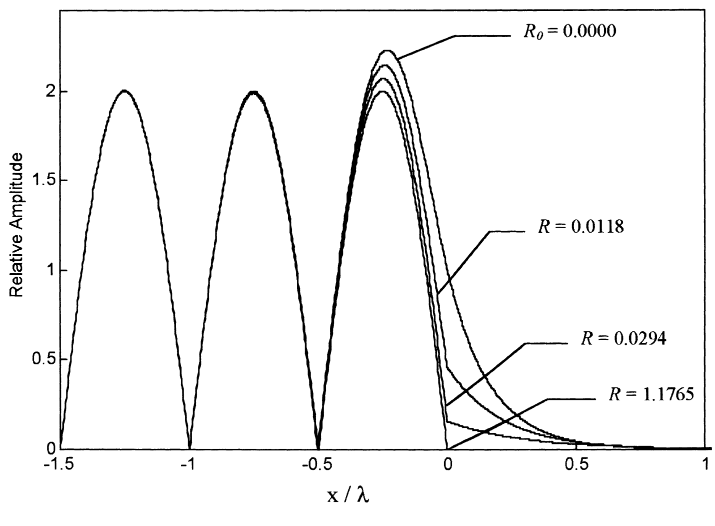

Fig. 5. Wave control using a single control force at $x = 0$. As R increases, the controlled wave differs from the Euler–Bernoulli solution.

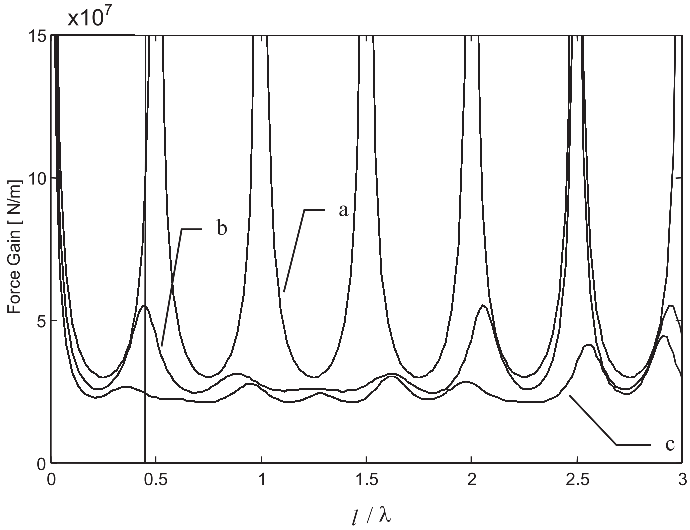

Fig. 6. Dependence of the modulus of control force gain g as a function of interval between the forces for (a) two forces controlling two waves, (b) three optimal forces controlling two waves ($l_3/l_2 = 1.2$) and (c) four optimal forces controlling two waves ($l_3/l_2 = 1.2$ and $l_4/l_2 = 1.55$).

The proposed control criterion does not consider any condition for a point inside the array, $l_1 > x > l_4$. This will not be an important restriction if the interval l between the two forces is conveniently chosen.

In Fig. 6(a), it can be seen how the modulus of the control force gain vector $g = |\{f\}|$ (called simply *force gain*) varies as a function of the ratio l/λ for a two force array.

A particular control situation arises when $R > 1$. In this case, a single external force can cause two different propagating wave disturbances, which can still be controlled with the four force (load) array. Here, the disturbance would be

$$\{D\} = \{A_{\text{in}2} \quad A_{\text{in}1} \quad 0 \quad 0\}^{\mathrm{T}}. \tag{31}$$

Similar to what occurs for Euler–Bernoulli beams (Brennan et al., 1995) the amplitude of the necessary control forces increase strongly when the distance between the forces is near an integer number of half-wavelengths. Fig. 7 shows the control wave behavior for the same beam considered in Fig. 6 for a two force control.

For the same beam configuration, if the number of control forces is increased in order to eliminate more waves generated inside the array, the force gain g will increase, and larger forces would be needed. Fig. 8 shows the four approaches considered before and their controlled response. These results indicate that, frequently, there is little or no advantage in controlling more than two propagating waves, especially for waves with small wavelength that have short nearfields (Fig. 6).

5.3. Pairs of moments array

The pair of moments array actuator has a special appeal. It can be easily generated by sandwiches of piezoelectric materials or laterally offset linear actuators and does not require a reaction body or skyhook

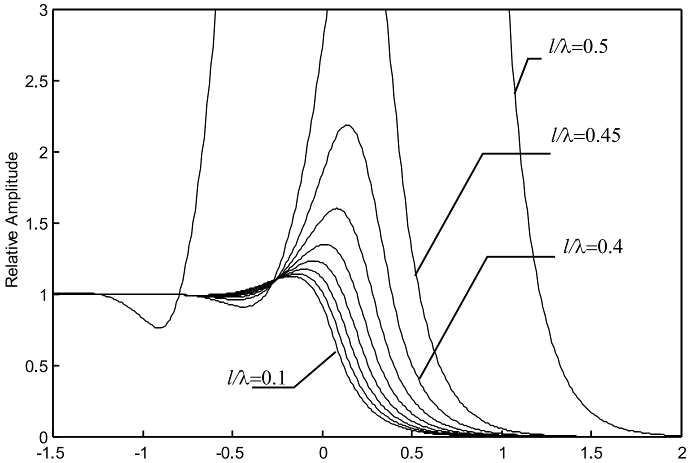

Fig. 7. The relative amplitude is plotted as a function of position ratio x/λ for different intervals λ between the two control forces in the array ($R = 8.7 \times 10^{-6}$).

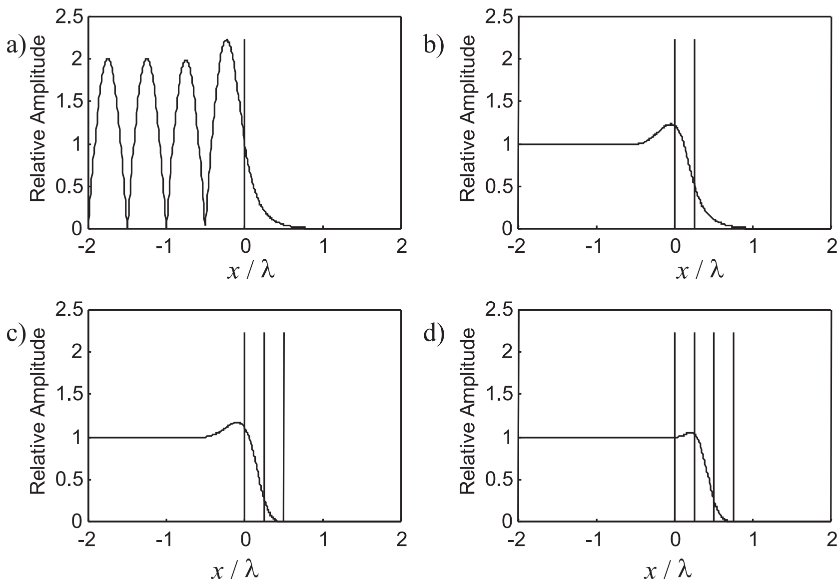

Fig. 8. Arrays with one to four control forces applied to the same beam considered before. The respective modulus of the force gain g are (a) for one control force applied, $g = 4.25 \times 10^7$ N/m, (b) for two control forces applied, $g = 5.12 \times 10^7$ N/m, (c) for three control forces applied, $g = 1.04 \times 10^8$ N/m, and (d) for four control forces applied, $g = 2.24 \times 10^8$ N/m.

for actuator anchoring. Fig. 9 shows such a representation. As a matter of fact, the condition of autonomous equilibrium is $\sum M = 0$. An array made of pairs of moments is just a particular and more restrictive case.

Following the same procedure adopted for the forces, Eq. (24) is still valid. Defining $\{B\}_i$ as a load vector due to a unit moment and zero force applied,

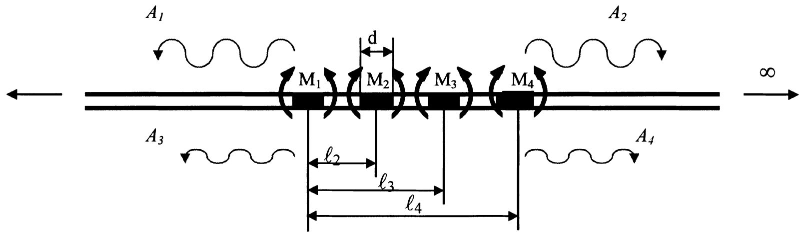

Fig. 9. Four pairs of moments array applied to an infinite beam and the waves generated by these moments out of the array. For simplicity, in this figure, l_1 was made zero.

$$\{B\}_i = \left\{ \begin{array}{c} 0 \\ 0 \\ (M/EI) \\ 0 \end{array} \right\}. \tag{32}$$

Then, the control wave generated by the four pairs of moments array applied in l_1, l_2, l_3 and l_4 would be likewise

$$\{A\}_{\text{control}} = M_1\{A\}_1 + M_2\{A\}_2 + M_3\{A\}_3 + M_4\{A\}_4. \tag{33}$$

Using the same control criteria as in the force case, the same four levels of fulfillment can be adopted. For the last and more general one,

$$\{A\}_{\text{control}} = \sum_{i=1}^{4} M_i\{A\}_i = -\{D\} = \{0 \quad -1 \quad 0 \quad 0\}^{\mathrm{T}}. \tag{34}$$

Writing Eq. (34) in matrix form and considering $\{D\}$ as a unit vector

$$\left[\{A\}_1\{A\}_2\{A\}_3\{A\}_4\right]\{M\} = \{0 \quad -1 \quad 0 \quad 0\}^{\mathrm{T}}, \tag{35}$$

where

$$\{M\} = \{M_1 \quad M_2 \quad M_3 \quad M_4\}^{\mathrm{T}} \tag{36}$$

Therefore, the control pairs of moments $\{M\}$ can be evaluated by

$$\{M\} = [A]^{-1} \left\{ \begin{array}{c} 0 \\ -1 \\ 0 \\ 0 \end{array} \right\}. \tag{37}$$

In Fig. 10, the results of such a control applied on a beam can be seen.

If less than four control conditions are imposed, i.e., if only a part of the generated waves are controlled, the number of control moments would be reduced analogously to what happened with force arrays.

As in the force case, the proposed control criterion does not impose any condition for a point inside the array, $l_1 < x < l_4$.

Letting d be the distance between two moments that form a pair, the control of a Timoshenko beam is plotted in Fig. 11 for various ratios d/λ.

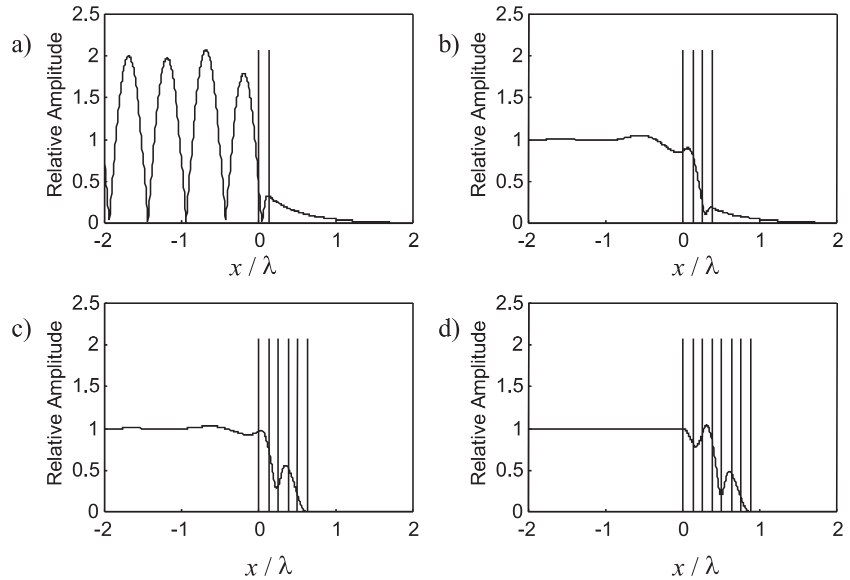

Fig. 10. Arrays with one to four pairs of moments applied to the same beam considered before. The respective moment gains are (a) for one pair of control moments applied, $g = 1.736 \times 10^9$ N, (b) for two pairs of control moments applied, $g = 1.227 \times 10^9$ N, (c) for three pairs of control moments applied, $g = 1.228 \times 10^9$ N, and (d) for four pairs of control moments applied, $g = 1.435 \times 10^9$ N.

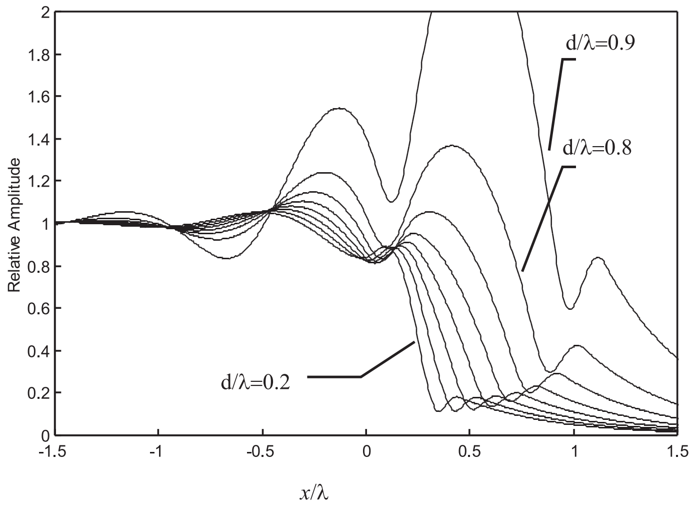

Fig. 11. The control of a Timoshenko beam by two pairs of moments for different ratios d/λ. The distance l between two pairs of moments is fixed as 0.25λ.

Another possibility investigated was the condition $d > l$. The main differences among the two situations arise inside the array. No particular advantage could yet be observed with such an arrangement.

The moduli of the control moments increase when d approaches an integer multiple of λ, as shown in Fig. 12, for any number of pairs of control moments. These controllability results can guide the pairs of moments array design when sufficiently narrow band disturbances are involved.

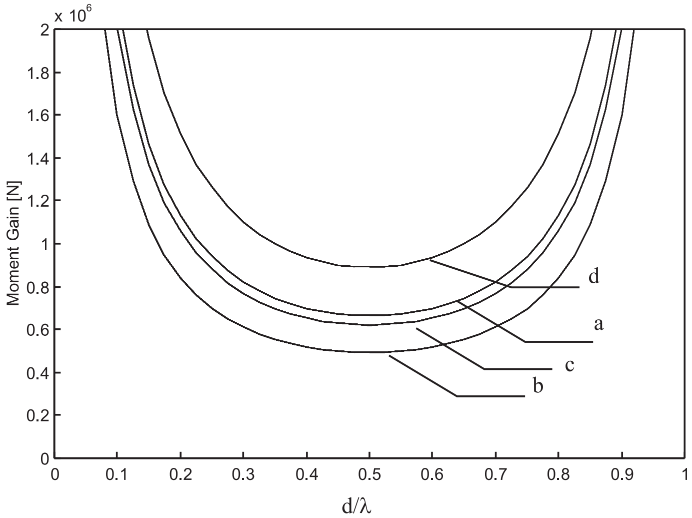

Fig. 12. Moment gain as a function of the ratio d/λ, for $l = 0.2\lambda$ and $R = 0.087$: (a) one pair of moments, (b) two pairs of moments, (c) three pairs of moments, and (d) four pairs of moments.

5.4. Minimum force (load) suppression control

In the control strategy adopted in Section 5.2, if less than four control conditions are imposed, only a part of the generated waves is controlled and the number of control loads would be reduced in the same proportion. It was also stated that, in many applications, the control of a pair of propagating waves could suffice.

A new and convenient strategy now proposed is the minimum force/load suppression control strategy, exemplified with two controlled waves and arrays of three instead of two loads. One additional condition imposes that the sum of the load moduli be minimal so that the three loads can be evaluated. The problem with equality constraints can be solved with Lagrangian multipliers. Taking the force array for simplicity, let F be the function

$$F = f_1 \cdot f_1^* + f_2 \cdot f_2^* + f_3 \cdot f_3^*, \tag{38}$$

where f_i^* is the complex conjugate of f_i. F is to be minimized and submitted to two constraints arising from Eq. (28):

$$G_1 = A_{11}f_1 + A_{12}f_2 + A_{13}f_3 + 1 = 0, \tag{39}$$

and

$$G_2 = A_{21}f_1 + A_{22}f_2 + A_{23}f_3 = 0. \tag{40}$$

Defining

$$H\{q\} = f_1 \cdot f_1^* + f_2 \cdot f_2^* + f_3 \cdot f_3^* - \alpha_1 G_1 - \alpha_2 G_2, \tag{41}$$

and imposing the minimum condition

$$\frac{\mathrm{d}H}{\mathrm{d}\{q\}} = \{0\}, \tag{42}$$

lends the linear system

$$[D]\{q\} = \{0 \quad 0 \quad 0 \quad -1 \quad 0\}^{\mathrm{T}}, \tag{43}$$

where

$$[D] = \begin{bmatrix} 2 & 0 & 0 & -A_{11} & -A_{21} \\ 0 & 2 & 0 & -A_{12} & -A_{22} \\ 0 & 0 & 2 & -A_{13} & -A_{23} \\ A_{11} & A_{12} & A_{13} & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 \end{bmatrix} \tag{44}$$

and

$$\{q\} = \{f_1 \quad f_2 \quad f_3 \quad \alpha_1 \quad \alpha_2\}^{\mathrm{T}}. \tag{45}$$

Eq. (43) is then solved to determine the control forces f_1, f_2 and f_3.

Up to this point, the adopted approach assumes a harmonic disturbance with frequency ω. The associated wavelength depends on this frequency among other physical characteristics (see Eq. (10)). A more general disturbance has a wider frequency spectrum. Therefore, with the purpose of avoiding the uncontrollable condition $l/\lambda = 0.5$, the array design could be arranged in such a way that the interval, l, between two control loads would not be fixed.

In other words, the three loads can be arranged to avoid the node's neighborhood at any frequency in a wider spectrum. In this way, the control will be effective in a wider range of frequencies. In fact, as shown in Fig. 6(b), the modulus of the control force gain is very large only when both intervals between the loads approach an integer multiple of the half-wavelength of the disturbance propagating wave. This strategy can be extended to more than three force array. Fig. 6(c) shows the gain modulus for a four force array controlling two waves. Fig. 13 compares the two force suppression control with the minimum force suppression control for three and four control force array. In all the three cases, just two propagating waves are controlled.

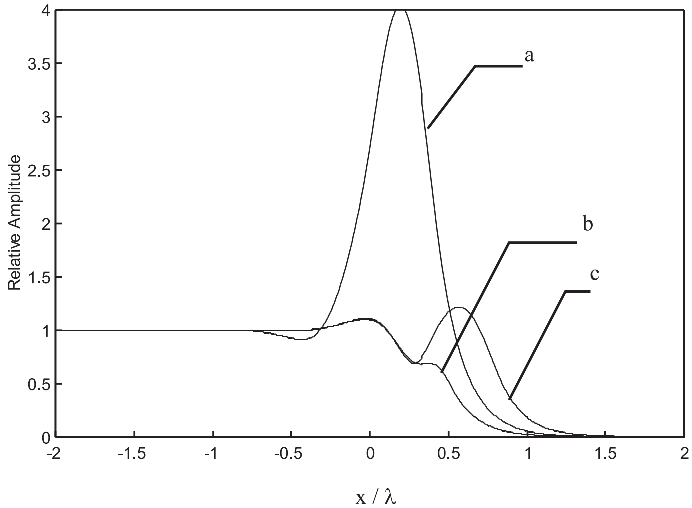

Fig. 13. Comparison between two strategies applied to the same beam of Figs. 6 and 7 with $l/\lambda = 0.45$ (a) two forces suppression control, (b) minimum force suppression control with three control forces ($l_3/l_2 = 1.2$), (c) minimum force suppression control with four control forces ($l_3/l_2 = 1.2$ and $l_4/l_2 = 1.55$).

This control strategy can be applied in a similar way to any situation where the number of loads is greater than that of the controlled waves, but three loads and two controlled waves seam to be the most advantageous one. The same approach with pairs of moments, instead of force, shows similar good results.

6. Conclusions

Timoshenko beam equation has been reviewed, and its wave solution for an infinite beam forced by a concentrated applied load was obtained and analyzed. The active control by a classical wave suppression strategy was implemented for two kinds of loads: force and pair of moments. A minimum load strategy is proposed, with the number of control loads exceeding the number of controlled waves. This control strategy needs no information about the disturbance source positions or about the transfer receptance between these points and the control load array as it is needed in the power-based strategies (Brennan et al., 1995). In practical engineering projects, these data are not always available. The disturbances are not always concentrated and often come from another structure.

Minimum force suppression control strategy (with three or more loads to control two waves) has been developed and presented a very good performance in the active control of a Timoshenko beam, as compared with standard suppression strategy. This advantage is specially important when the excitation disturbances have a wide frequency spectrum. The new strategy proposed can potentially result in smaller actuators, although in greater number.

In practical applications, due to the usually low efficiency and highly nonlinear behavior of exciters, the gross power demanded by the control is roughly proportional to the force (loads) required and not to the net power demanded. Thus, minimum force suppression control strategy may also lead to small gross power requirements, although the net power needed by this strategy could be quite different from that of minimum power strategy.

When compared with the power-based strategies, the minimum load suppression control strategy proposed has a local approach that does not need to take account of the beam structure, except between the sensors and the load array.

However, stated here for the infinite beam, the approach can be readily applied to finite lengths. The same load array may simultaneously control waves propagating in both directions, provided they are adequately measured by sensor arrays.

References

Brennan, M.J., Elliott, S.J., Pinnington, R.J., 1995. Strategies for the active control of flexural vibration on a beam. Journal of Sound and Vibration 186 (4), 657–688.

Carvalho, M.O.M., 1998. Active control of waves on a Timoshenko beam D.Sc. Thesis, COPPE/UFRJ, Rio de Janeiro, Brazil (in Portuguese).

Chen, S.H., Wang, Z.D., Liu, X.H., 1997. Active vibration control and suppression for intelligent structures. Journal of Sound and Vibration 200 (2), 167–177.

Corn, S., Bouhaddi, N., Piranda, J., 1997. Transverse vibration of short beams: finite element model obtained by a condensation method. Journal of Sound and Vibration 201 (3), 353–363.

Elliott, S.J., Billet, L., 1993. Adaptive control of flexural waves propagating in a beam. Journal of Sound and Vibration 163 (2), 295–310.

Farghaly, S.H., Gadelrab, R.M., 1995. Free vibration of a stepped composite Timoshenko cantilever beam. Journal of Sound and Vibration 18 (5), 886–896.

Gardonio, P., Elliott, S.J., 1996. Active control of waves on a one-dimensional structure with a scattering termination. Journal of Sound and Vibration 192 (2), 701–730.

Gibbs, G.P., Fuller, C.R., 1992. Excitation of a thin beam using asymmetric piezoelectric actuators. Journal of the Acoustical Society of America 92 (6), 3221–3227.

Hagedorn, P., Schmidt, J., 1989. On active vibration damping in beams. Proceedings of the first Pan-American Congress on Applied Mechanics, Rio de Janeiro, January, pp. 656–658.

Hirami, N., 1997. An active maximum power absorber for the reduction of noise and vibration. Journal of Sound and Vibration 200 (3), 261–279.

Houmat, A., 1995. Vibrations of Timoshenko beams by variable order finite element. Journal of Sound and Vibration 187 (5), 841–849.

Karlson, S.E.S., 1996. A computational device for incident waves to a structure via a Rayleigh–Timoshenko beam. Journal of Sound and Vibration 190 (5), 739–750.

Lueschen, G.G.G., Bergman, L.A., MacFarland, D.M., 1996. Green's function for uniform Timoshenko beams. Journal of Sound and Vibration 194 (1), 93–102.

Mace, B.R., 1984. Waves reflection and transmission in beams. Journal of Sound and Vibration 97 (2), 237–246.

Mace, B.R., 1987. Active control of flexural vibrations. Journal of Sound and Vibration 114 (2), 253–270.

Mead, D.J., 1994. Waves and modes in finite beams: application of the phase-closure principle. Journal of Sound and Vibration 171 (5), 695–702.

Nagaya, K., 1995. Method of control of flexible beams subjected to forced vibrations by use of inertia cancellations. Journal of Sound and Vibration 184 (2), 184–194.

Pan, X., Hansen, C.H., 1995. Active control of vibratory power transmission along a semi-infinite plate. Journal of Sound and Vibration 184 (4), 585–610.

Tong, X., Tabarrok, B., Yeh, K.Y., 1995. Vibration analysis of Timoshenko beams with non-homogeneity and varying cross-section. Journal of Sound and Vibration 186 (5), 821–835.